A Generalized Method to Optimize Acoustic Intensity Field by Using Source Array

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Abstract

Acoustically bright zone is defined as a zone where the listener can acquire better sound quality than others. The acoustically bright zone can be generated by improving a desired acoustic variable on a selected zone, and the ‘zone control’ can be done by controlling multiple sources. Among many possible definitions on the acoustic variables, acoustic intensity is tried as an objective function to enhance the sound power radiation into the listening area.

In previous work, acoustic intensity projected to a direction is considered as object function [J.-W. Choi and Y.-H. Kim, “Acoustic intensity optimization using source array,” in Proc. Inter-noise 2003, N777], so that one can maximize acoustic intensity component propagates to the desired direction. This approach shows that acoustic intensity field can be manipulated into a desired direction using a small number of sources, but it sometimes fails to control the direction of intensity. Extending this work, this paper presents a generalized method that can also manipulate the propagating direction of wave front. By employing normalized transfer functions between the multiple sources and measurement points during the optimization process, the directional characteristics of intensity field can be improved.

1. Introduction

Sound power or sound intensity is one of major factors used to evaluate performance of acoustic source. The conventional acoustic source is constituted of single actuator, and the performance measured by how well it radiates sound power. However, from the listeners’ point of view, what is really important is sound power transmitted to the distinct region where the listener is located. With this initiative, we propose a method to maximize sound power at a surface we define(Fig. 1(a)). Additionally, we attempt to control its direction as well as the magnitude of sound intensity in a region(Fig. 1(b)).

To control sound field for the region that is defined or selected, it is necessary to manipulate multiple sources. In the previous papers[1-3], we have presented several methods to generate acoustically bright zone using multiple sources. The method of controlling sound intensity of a zone, can be regarded as extension of this work. We, first briefly review present method[3] of intensity field control. The method has to do with determining optimal input signals of multiple sources that maximizes acoustic intensity to a desired direction. The second deals with how the proposed method has to be modified to enhance directional characteristics of power flow.

2. Problem Definition

2.1 Mathematical formulation

Let’s consider multiple control sources, which is fixed in space bounded(or unbounded) by arbitrary boundary condition. We don’t impose any restrictions on the location and radiation characteristics of control sources
sources. Then we denote active intensity at the position $\mathbf{r}$ as

$$\tilde{I}_a(\mathbf{r}) = \frac{1}{2} \text{Re}[P(\mathbf{r}) \tilde{U}(\mathbf{r})^*],$$

(1)

where $P(\mathbf{r})$ and $\tilde{U}(\mathbf{r})$ are complex magnitude of pressure and velocity of pure tone sound at the position $\mathbf{r}$.

Final goal of this work is to maximize overall acoustic intensity within a defined zone to a desired direction. We call this zone as control zone $V_0$. With this purpose, active intensity averaged throughout the zone $V_0$ is accepted as a measure of zone’s overall intensity. Then the performance of multiple sources is evaluated in terms of average intensity projected to a control(desired) direction, $\mathbf{n}_G$.

$$\mathbf{V}_{\alpha} \mathbf{H} = \mathbf{C}_q \mathbf{q},$$

(2)

The desired direction $\mathbf{n}_G$ is a function of space, but for simplicity, we assume that it is constant throughout the zone.

Denoting the transfer function between multiple sources and field position $\mathbf{r}$ as

$$H(\mathbf{r} | \mathbf{r}_s) = \left[H(\mathbf{r} | \mathbf{r}_s^{(1)}), \ldots , H(\mathbf{r} | \mathbf{r}_s^{(K)})\right]$$

and control signal supplied to each source as

$$p(\mathbf{r}) = H(\mathbf{r} | \mathbf{r}_s) q(\mathbf{r}_s),$$

then the pressure and velocity of Eq. (1) can be expressed in terms of control signal

$$\tilde{U}(\mathbf{r}) = \frac{1}{\rho_0 c_0} V H(\mathbf{r} | \mathbf{r}_s) q(\mathbf{r}_s).$$

(6)

Then the space-averaged active intensity of Eq. (2) can be rewritten as follows:

$$\langle \tilde{I}_a, \mathbf{n}_G \rangle_{V_0} = \frac{1}{2 \rho_0 c_0} \mathbf{q}^H \mathbf{C}_q \mathbf{q},$$

(7)

$$\mathbf{C}_q = -\text{Im} \left[ \frac{1}{k V_0} \int (\mathbf{n}_G \cdot \nabla H)^H H dV \right]$$

where $H$ and $\mathbf{q}$ are abbreviations of $H(\mathbf{r} | \mathbf{r}_s)$ and $q(\mathbf{r}_s)$.

3. Solution method

3.1 Definition of cost function

To maximize overall acoustic intensity using limited input power of multiple sources, we introduce constraint on the input power $J_0 = \mathbf{q}^H \mathbf{q}$, and the cost function of this problem can be defined as follows.

$$\text{maximize } \alpha = \frac{\langle \tilde{I}_a, \mathbf{n}_G \rangle_{V_0}}{J_0} = \frac{\mathbf{q}^H \mathbf{C}_q \mathbf{q}}{H^2 \mathbf{q}^H \mathbf{q}}$$

(8)

Where $J_0 = \frac{1}{2 \rho_0 c_0} H^2 \mathbf{q}^H \mathbf{q}$, and $H_0$ is a normalization constant.

Eq. (8) essentially maximizes absolute magnitude of overall active intensity projected to the desired direction, using finite control effort $\mathbf{q}^H \mathbf{q}$. The control effort can be thought as a penalty of maximization problem.

To practically implement this procedure, the transfer function $H$ and its gradient $\nabla H$ has to be identified. There are several methods. For example, a direct measurement[3] is applicable when the zone size is small compared to the wavelength. On the contrary, if the zone is greater than the wavelength or is given as a volume with closed surface, then measurement points can be reduced by using boundary element method(BEM) on the surface of the volume and predicting interior sound field(For further information on the BEM formulation of sound power, cf. [4,5]).

The solution of Eq. (8) can be easily obtained by eigenvalue analysis.

$$\mathbf{C}_q \mathbf{q}_{opt} = \alpha H_0^2 \mathbf{q}_{opt}$$

(9)

That is, an eigenvector corresponds to the maximum eigenvalue of matrix $\mathbf{C}_q$.

What is shown in the next example is the result of numerical simulation performed in 3-D free field. The control zone $V_0$ is configured as a square 2-D plane of finite aperture($L_0 = 0.55 \lambda$), and 5 monopole control sources are employed as the control sources.

Figure 3(a) shows configuration of control sources and control zone. In this example, the sources are located at the same distance from the origin($R_0 = 3 \lambda$), and angular distances between the sources are also equal. Control direction $\mathbf{n}_G$ is set to be upward direction ($\phi_0 = 90^\circ$), and when we performed optimization given by Eq. (8), the result of optimized intensity field shows good match with the control direction.
However, one should note that the cost function of Eq.(8) only evaluates the performance in terms of absolute magnitude of intensity. This means, it can not manifest intensity direction.

The next example elaborates this problem. The control sources are located at different distances from the origin \( R = [1.2, 2.4, 3.6, 4.8, 6.0] \lambda \), counterclockwise).

We can observe that the direction of power flow does not match to what we intended(Fig. 4(b)). This is because the sources near the control zone generate more intensity magnitude(even it is projected) than others with the same input power, although its direction is somewhat different from the control direction.

The change of cost function with respect to the control direction is presented in Fig. 4(c). It reveals that the optimization is heavily effected by the control direction.

3.2 Modification for direction control

Therefore, if we are more interested in the direction of power flow than the magnitude itself, the cost function has to be modified so that the direction of intensity has more priority than the magnitude.

Note that the acoustic intensity can be interpreted as a product of acoustic potential energy and rate of phase change(for example, [6]).

\[
\vec{I}_e(\vec{r}) = \frac{1}{2\rho_0 c} \left| P(\vec{r}) \right|^2 \nabla \phi(\vec{r})
\]

where \( P(\vec{r}) = |P(\vec{r})| e^{i\phi(\vec{r})} \). Propagating direction of sound power only depends on the spatial phase change. Therefore, by normalizing transfer function of each source, we can decrease the effect of acoustic potential energy and impose more priority to the wave front propagation. The modified cost function can be defined as follows,

\[
\alpha' = \frac{q^H C_{eq} q}{\left( q^H \text{Diag} \left[ \int H^H H dV / 2\rho_0 c V_0 \right] q \right)}
\]

where \( \text{Diag}[A]_{nm} = A_{nm} \delta_{nm} \).

The denominator of Eq. (11) is proportional to the sum of potential energy generated by each source. The normalization constant is now changed so that the penalty function(denominator of Eq.(11)) of each source increases according to the potential energy they generates.
As stated in Eq. (9), optimal solution of Eq. (11) is determined as the maximum eigenvector of the following matrix (for example, cf. [7]).

\[(E_{ij}^p)^\dagger C_{ij}^{opt} q_{ij}^{opt} = q_{ij}^{opt}\] (12)

In most cases we don’t have to worry about the matrix inversion, because it is singular only when a redundant source exists, i.e., a source does not generate any pressure in \(V_0\). In that case, it is natural to discard the singular (or close to singular) sources from the control procedure.

Figure 5 is a result of the modified optimization. On the contrary to Fig. 4, we can observe that the direction of each intensity vector is improved. Source strength of each control source shows that sources radiating sound to similar direction to the control direction (4th and 5th source of Fig. 5(a)) have higher source strength than others. Compared to the original case, the modified cost function is robust, i.e., has uniform performance in all control direction.

4. Conclusion

A method to control the direction of sound power in a defined region is discussed. From the original method to maximize sound power using finite control effort, modified cost function that is more focused on the direction of intensity field is proposed.

Numerical simulations with scattered source distribution show that the modified approach gives better directional characteristic and robust with respect to the control direction.

5. References


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