Compensation of Parametric-Effect-Induced Error in Airborne Ultrasound Doppler Velocimetry

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Abstract

A non-contact vibration measuring technique based on ultrasonic Doppler effect is studied. Two effects induce phase modulation on the received ultrasound signal, which are Doppler effect and nonlinear parametric effect. With the increasing of vibrating frequency, the phase variation due to parametric effect is getting greater and greater. In this paper, by measuring the acoustic pressure generated by the low frequency vibration, the parametric effect is taken into account for estimating the vibrating velocity. The results show that it is better than the method of only taking account of the Doppler effect especially as the vibrating frequency increases. For example, for a vibrating body of diameter 10 cm at frequency of 1.5 kHz, the error of velocity measurement with parametric-effect-compensation is getting less than one half of that without compensation.

1. Introduction

Vibration measurements are important in many applications. Most often sensors such as accelerometers are used with excellent results. However, in some applications it is not possible to attach a measuring device directly onto the vibrating structure. It can be due to shortage of time or due to the weight of the sensor. A complicated surface structure may also decrease the possibilities to accomplish the measurement. In these cases, non-contact methods have to be chosen [1].

In this paper, a non-contact ultrasonic system for vibration measurement based on the same principle as optical Doppler systems is studied. In this system, a transmitting ultrasonic transducer sends the ultrasonic beam into air. The beam is reflected by the surface investigated, and then travels to a receiving ultrasonic transducer. When the surface investigated is moving, the routine length of ultrasonic beam from transmitter to receiver is modulated by the movement of surface investigated. As result, the phase of ultrasonic beam is modulated. This is Doppler effect. At the same time, the vibrating surface is emitting acoustic wave into air. Due to the inherent nonlinearity of acoustic basic equation, the acoustic wave interacts with the ultrasonic beam. These interactions induce a phase shift of ultrasonic beam too. This is nonlinear parametric effect. The phase shift caused by these effects is detected by a coherent electronic circuit. If one wants to determine the vibration characteristics of the scanned surface from the phase shift of high frequency ultrasonic beam, the nonlinear parametric effect should be taken into account instead of only taking Doppler effect into account.

Because the low frequency acoustic wave emitted by vibrating body is not plane wave in fact, the influence of parametric effect can not just base on plane wave assumption. In this paper, by measuring the acoustic pressure generated by the low frequency vibration, the parametric effect is taken into account for estimating the vibrating velocity. The results show that it is better than the method of only taking account of the Doppler effect especially as the vibrating frequency increases. For example, for a vibrating body of diameter 10 cm at frequency of 1.5 kHz, the error of velocity measurement with parametric-effect-compensation is getting less than one half of that without compensation.

2. Principle

Figure 1 shows the basic arrangement of the ultrasonic Doppler vibrometer. A piezoelectric transducer transmits a high frequency continuous ultrasonic wave onto the moving surface. The reflected wave is detected by another piezoelectric transducer. Two effects induce phase modulation on the ultrasonic wave signal: Doppler effect and parametric effect.

2.1. Doppler effect

The ultrasonic transmitter emits an ultrasonic beam toward a vibrating surface. As the investigated surface is vibrating, the path length of ultrasonic wave from the transmitter to the reflector (vibrating surface) and then
back to the receiver is varying. That causes the variation of round trip time and so on the variation of received signal phase. The phase variation caused by the Doppler effect is directly proportional to the vibrating surface displacement. According to Doppler effect, the received signal is

$$s(t) = \frac{A_0}{K} \cos\left(\omega_0 t - \psi - \varphi_D\right)$$

(1)

where \( \varphi_D \) is the phase shift caused by Doppler effect,

$$\varphi_D = \frac{2A_t}{c_0} \omega_0 \cos \theta \sin\left(\omega_2 \left(t - \frac{L}{c_0}\right)\right)$$

(2)

\( A_0 \) and \( \omega_0 \) are the amplitude and the angular frequency of the transmitted ultrasonic wave beam, \( K \) is a constant determined by the characteristic of medium and the distance between transducers and vibrating body, \( c_0 \) is the sound velocity, \( \psi \) is a constant phase term, \( A_t \) and \( \omega_2 \) are the amplitude and the angular frequency of the vibration, \( L \) is the distance between the transducer and the vibrating surface, \( \theta \) is the angle between the propagation direction and the perpendicular of the vibrating surface.

2.2. Parametric Effect

The vibrating surface emits a low frequency acoustic wave into the air. In the air, this low-frequency acoustic wave and the high-frequency ultrasonic wave interact with each other. Physically the phase modulation of a high-frequency wave is caused by the variation of the propagation velocity of points of its profile in the field of the low-frequency acoustic wave.

The variation of the propagation velocity in the interaction zone depends on two factors [2][3]:

The first is the excess pressure caused by low frequency sound.

$$\Delta c_1 = \frac{\gamma - 1}{2\rho_0 c_0^2} p_L'$$

(3)

where \( \gamma \) is the ratio of the specific heats, \( \rho_0 \) is the equilibrium density of air, \( p_L' \) is the instantaneous pressure value of the low frequency sound field.

The second is the particle velocity of the medium:

$$\Delta c_2 = u'_L \cos \theta$$

(4)

where \( u'_L \) is the instantaneous particle velocity of air.

The resultant velocity increment of points on the high frequency ultrasonic wave profile is the sum \( \Delta c = \Delta c_1 + \Delta c_2 \). The resultant velocity variation \( \Delta c \) can be considered as a local phase shift created by the pressure and the particle velocity of the low frequency sound field. The global effect on the high frequency wave along the path \( L \) between the emitter and the receiver is simply obtained by integration. In this paper, only pressure is measured for compensation.

The parametric effect is present on both trajectories:

- From the transmitter to the vibrating surface

Assuming that the instantaneous pressure at point A on the routine from transmitter to vibrating surface is

$$p_L' = p_L \cos(\omega_1 t + \phi_L)$$

(5)

where \( p_L \) and \( \phi_L \) are the pressure amplitude and phase of low frequency sound field at this point. The distance between point A and the transmitter is \( x \).

The resultant velocity increment at this point due to the excess pressure caused by low frequency sound is

$$\Delta c = \mu p_L'$$

(6)

where \( \mu \) is a coefficient constant depending on the characteristics of air.

The phase shift of high frequency ultrasonic wave caused by the interaction between ultrasonic beam and low frequency acoustic wave at the neighborhood of this point is

$$\Delta \phi = -\frac{\omega_2}{c_0^2} \Delta c \Delta x$$

(7)

It follows Eq. (5), (6) and (7) then

$$\Delta \varphi = -\frac{\mu \omega_2}{c_0^2} p_L$$

$$x \cos(\omega_1 t + \phi_L - k_L (2L - x)) \Delta x$$

(8)

where \( k_2 \) is the wave number of low frequency acoustic wave.

- From the vibrating surface to the receiver

Assuming that the instantaneous pressure at point B on the routine from transmitter to vibrating surface is the same as Eq. (5). The distance between point B and the receiver is \( x \). The phase shift of high frequency ultrasonic wave caused by the interaction between ultrasonic beam and low frequency acoustic wave at the neighborhood of this point is

$$\Delta \varphi = -\frac{\omega_2}{c_0^2} p_L \cos(\omega_1 t + \phi_L - k_L x) \Delta x$$

(9)

Assuming the acoustic wave emitted from vibrating surface is a plane wave and the pressure at transmitter and receiver is \( p_L \cos(\omega_1 t) \), then

$$p(x) = p_L \cos(\omega_1 t + k_L x \cos \theta)$$

(10)

For the case \( \theta = 0 \), the global phase shift is

$$\Delta \phi = -\frac{2 \mu \omega_2}{c_0^2} p_L \times$$

$$\left(\frac{\sin(k_L L)}{k_L} \cos(\omega_1 t - k_L L) + L \cos(\omega_1 t)\right)$$

(11)
In the case that the low frequency acoustic is a plane wave, the pressure is

\[ p_L = \rho_0 c_0 \omega_L A_L \]  

(12)

For this case, compensation for parametric-effect-induced error can be done by only multiplying a coefficient to the output of FM demodulator. In fact, the pressure is not only dependent on vibrating velocity. It also has relation with the area of vibrating surface. As the area of vibrating surface becomes larger, the pressure of low frequency acoustic wave emitted from this surface becomes higher. At this situation, parametric-effect-induced error should be compensated by measuring the pressure of low frequency acoustic wave. From Eq. (11), parametric-effect-induced error can be described as

\[ \Delta \rho = \xi p_L \cos(\omega_L t + \psi_0 + \psi_1) \]  

(13)

where \( p_L \) is the amplitude of low frequency acoustic pressure at a fixed point, \( \psi_0 \) is the phase of low frequency acoustic pressure at that point, \( \xi \) is a amplitude correction coefficient dependent on the position of pressure measuring point, \( \psi_1 \) is a phase correction coefficient dependent on the position of pressure measuring point and vibrating frequency.

3. Experiment

The experimental setup of airborne ultrasound vibration measurement system with compensation of parametric-effect-induced error is illustrated in Fig. 2. The vibrating object is an aluminum plate of diameter 5 cm or 10 cm driven by EMIC model 512-D/A shaker. The microphone used for pressure measurement is a B&K type 4136 condenser microphone. A directive 200 kHz ultrasonic beam toward the vibrating aluminum plate is emitted. The ultrasonic beam is reflected by the aluminum plate and picked up by an ultrasonic receiver. The received signal is amplified and processed by using an FM demodulator to obtain the frequency shift, which is proportional to the velocity of moving surface. In this experiment, a phase-locked loop (PLL) demodulator is used as the FM demodulator.
The experiment results are shown in Fig. 3 and Fig. 4, where dashed lines are results without compensation, solid lines are results with compensation, vertical ordinate is relative sensitivity $S_r = \frac{\text{Sensitivity}}{\text{Sensitivity}(D = 5 \text{ cm}, f = 500 \text{ Hz})}$. When we calculate the vibrating velocity from the output of FM demodulator taking account the Doppler effect only (without compensation), we notice small error at low frequency for vibrating body with small surface area and an increasing error with increasing frequency and increasing surface area. If we calculate the vibrating velocity from the outputs of FM demodulator and microphone (with compensation), we find that the measurement error becomes smaller than that without compensation, especially when vibrating frequency is higher than 1 kHz for large vibrating body. For example, for a vibrating body of diameter 10 cm at frequency of 1.5 kHz, the error of velocity measurement with parametric-effect-compensation is getting less than one half of that without compensation.

4. Conclusions

In this paper, by measuring the acoustic pressure generated by the low frequency vibration, the parametric effect is taken into account for estimating the vibrating velocity. The results show that it is better than the method of only taking account of the Doppler effect especially as the vibrating frequency increases. For example, for a vibrating body of diameter 10 cm at frequency of 1.5 kHz, the error of velocity measurement with parametric-effect-compensation is getting less than one half of that without compensation.

5. References