Modeling the steady and unsteady operation of a piezo-electric bar type motor

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Abstract

While piezo-electric traveling wave motors have been examined in detail in the literature, other types of piezo-electric drives, such as CANON’s bar type motor (or wobbling-disk motor) have received far less attention so far. In this paper the derivation of the equations for steady and unsteady operation mode are outlined. The equations of motion are derived with the help of EULER angles and EULER’s equations. For normal operation of this motor it is assumed that the contact condition between stator and rotor will be that of pure stick, only. Rotor and stator are therefore coupled by a kinematic constraint. The excitation generated by the piezoceramic is described by two 90° phase-shifted harmonic excitation torques.

Steady and unsteady operation of the motor is studied for different working conditions and the onset of instability can also be studied using the equation presented in this paper.

1. Introduction

Several different miniature ultrasonic bar-type motors using bending modes of a rod have been presented in the literature. Okumura’s [1] prototype of a bar-type ultrasonic motor is being used today in CANON’s autofocus lenses, which are being mass produced for many years now. In 1992 the previously used ring-type motor was replaced by the bar-type motor (U1000) in the autofocus lenses. Since 1992 the bar-type motor went through different stages of enhancements. The type of motor which is used in today’s autofocus lenses since 2002 is CANON’s Micro USM. Excitation units which are applied to the ultrasonic motors U400 and Micro USM generate a λ/4-wave due to their arrangement of electrodes while type U1000 generates a λ/2-wave. Note that for a λ/4-wave stator and rotor have two contact zones while for a λ/2-wave it is one contact zone between the two bodies.

In this paper the ultrasonic bar-type motor first described in [1] is modeled to cover salient features. It seems that the same model may however also be used to analyze other ultrasonic motors such as mentioned by Kurosawa [2], Dong [3] or similar designs that are based on the same working principle, namely making use of the generated bending motion of the bar-type stator.

2. Motor model and working principle

The mathematical model presented in this paper corresponds to the λ/2-type (U1000) and rotor and stator are modeled as rigid disks having one contact point. A schematic representation of this rigid body model is depicted in Figure 1.

Figure 1: Rigid body model of the ultrasonic motor

The cylindrical stator is excited by means of single-layered piezoceramic actuators generating a rotating torque vector orthogonal to the geometrical axis of the stator. One torque is generated by a sinusoidal high frequency signal, while the other follows from a cosine signal, i.e. the two excitation inputs exhibit a phase-shift of 90°, so that the resulting steady state vibration of the stator yields a wobbling motion of the stator disk. Three
restoring torques act along the X-, Y- and Z-axes which are modeled with the help of three rotational springs. The elastic support along the Z-axis corresponds to an additional degree of freedom. In addition, damping is introduced in parallel to all springs. This mitigates numerical difficulties while solving the differential equations.

The working principle is based on the frictional interaction between stator and rotor. The operation frequency of the motor is 37 kHz which is close to the eigenfrequency of the stator at 36.4 kHz. The motor torques are defined by the friction forces (in particular the component in the circumferential direction) and the distance of the contact points to the rotational axis. The torques of ultrasonic motors thus depend on their dimensions and in particular by their driving radii. This can be verified from publications such as [2, 3, 1], where motors with different design parameters and in particular also different geometric dimensions are compared, and the highest stall torque is reached in the motor of the largest radius.

3. Mathematical Model

The equations of motion are derived using EULER angles (sequence 3-1-3). The position and velocity vectors of any point on the stator can be represented in terms of these angles. Table 1 summarizes all reference frames having used the rotating angles $\phi$, $\theta$, $\psi$ for the three rotations about $z$, $x_1$ and $z_2$, respectively.

<table>
<thead>
<tr>
<th>ref. frame</th>
<th>type</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>$XYZ$ inertial (NEWTONian)</td>
</tr>
<tr>
<td>A</td>
<td>$x_1y_1z_1$ 1st intermediate (nodal line frame)</td>
</tr>
<tr>
<td>B</td>
<td>$x_2y_2z_2$ 2nd intermediate</td>
</tr>
<tr>
<td>S</td>
<td>$xyz$ body-fixed</td>
</tr>
</tbody>
</table>

3.1. Kinematics

The position vector of the highest point of the stator (which is considered to be the contact point) is described in the 2nd intermediate frame as

$$ r_C = R e_{y_2} + H e_{z_2}. $$

(1)

The angular velocity vector of the stator is

$$ \omega = \dot{\psi} e_z + \dot{\theta} e_{x_1} + \dot{\phi} e_{z_2}. $$

(2)

Thus, the velocity of the contact point is given by

$$ v_C = \omega \times r_C. $$

(3)

If no slip occurs (which was a previously made assumption for this paper) between stator and rotor, the velocity of the contact point at the rotor’s surface is equal to $v_C$.

The rotor’s angular velocity is then obtained by

$$ \omega_r = \frac{(\omega \times r_C) \cdot e_{x_1}}{R^*}, $$

(4)

where $R^*$ is the distance from the rotor axis to the contact point on the rotor’s surface (see Figure 1). It can be verified that

$$ R^* = \sqrt{(r_C \cdot e_{x_1})^2 + (r_C \cdot e_{y_1})^2}. $$

(5)

3.2. Dynamics

The motion generated in the stator is passed on to the rotor via the forces and torques at the contact point. Thus, the equations of motion of stator and rotor are coupled through the dynamics at this point. From the free body diagram (Figure 2), the force vector at the contact point of the stator is

$$ F_{contact} = -f_n e_N - f_r e_{y_1} - f_c e_{z_1}, $$

(6)

where $f_n$, $f_r$, $f_c$ are the force components in normal, radial, and circumferential direction, respectively. The torque vector acting upon the stator is the vector sum of the excitation $M_E$, the restoring $M_R$, the damping $M_D$, and the contact torque vector $r_C \times F_{contact}$:

$$ M^{(O)} = M_E + r_C \times F_{contact} - M_R - M_D. $$

(7)
The excitation torque vector which describe the excitation originally generated by the piezoceramic is
\[
\mathbf{M}_E = m_X \mathbf{e}_X + m_Y \mathbf{e}_Y ,
\]
where \( m_X \) and \( m_Y \) are the input torques, one excited by a cosine and the other by a sine signal, respectively. The restoring torque is defined by
\[
\mathbf{M}_R = c_R \theta \mathbf{e}_z + c_t (\phi + \psi \cos \theta) \mathbf{e}_z ,
\]
with \( c_R \) and \( c_t \) are the rotating stiffness coefficients about the nodal line and the space-fixed \( Z \)-axis, respectively. The definition of the damping torque about the nodal line, in parallel to the restoring torque, does not describe the characteristic properly. In stationary motion, the angular velocity component in the direction of the nodal line is expected to be zero. Consequently the effect of the damping term would be lost, if not defining the damping torque in such way that
\[
\mathbf{M}_D = d_R \omega X \mathbf{e}_X + d_R \omega Y \mathbf{e}_Y + d_R \omega Z \mathbf{e}_Z .
\]

Additionally to the contact forces \( \mathbf{F}_{\text{contact}} \), a spring force \( \mathbf{f}_s \) and damping force \( \mathbf{f}_d \) act on the rotor disk (see Figure 2), with
\[
\mathbf{f}_s = (c_s z_R + f_0) \mathbf{e}_z \quad \text{and} \quad \mathbf{f}_d = d_s \dot{z}_R \mathbf{e}_z .
\]

The motion of the rigid disk of the rotor is specified by two generalized coordinates, namely, a rotation angle \( \phi_R \) and the coordinate \( z_R \) (from the equilibrium position) along the axial direction. The rotor equation describing the motion in the axial direction is
\[
m_R \ddot{z}_R + d_s \dot{z}_R + c_s z_R = f_n - f_0 \quad (14)
\]
and for the rotational motion of the rotor disk one has
\[
\frac{d}{dt}(I_R \omega_R) = T_{\text{load}} - f_x R^* \quad (15)
\]
\( m_R \) and \( I_R \) are the mass and moment of inertia of the rotor disk, respectively and \( T_{\text{load}} \) is the external loading torque. Under the assumption of contact and pure stick at all times (14) and (15) are substituted into the stator’s equations (12). The outcome by substituting and simplifying above’s equations is a system of \( 2 \times 3 \) first order differential equations which in matrix notation can be written as
\[
\mathbf{M}(\mathbf{q}) \dot{\mathbf{q}} = \mathbf{f}(\mathbf{q}, t) ,
\]
with the state vector \( \mathbf{q} = [\phi, \theta, \psi, \omega_x, \omega_y, \omega_z]^T \).

### 4. Steady State

It is convenient to start with the principle of angular momentum about the origin \( O \)
\[
\frac{d}{dt}(\mathbf{L}^{(O)}) + \mathbf{\Omega} \times \mathbf{L}^{(O)} = \mathbf{M}^{(O)} .
\]

In steady state the derivative of the vector of angular momentum \( \frac{d}{dt}(\mathbf{L}^{(O)}) \) with respect to reference frame \( A \) is zero. Table 2 contains the assumptions for stationary operation. Under stationary motion, \( \theta \) is constant and consequently \( \dot{\theta} = 0 \). Also, as a result \( \omega_R \) and \( z_R \) are constant, and \( \ddot{\omega}_R \), \( \dot{z}_R \), and \( \ddot{z}_R \) are therefore zero. Substituting and simplifying above’s assumptions into the equations of motion (16) yield a system of three algebraic equations of which only the first two are coupled. The nodal line rotates with \( \dot{\phi} = \Omega \) and the condition \( \omega_z = 0 \) leads to
\[
\dot{\psi} = -\frac{\dot{\phi}}{\cos \theta} .
\]

The system of algebraic equations reduces to one equation
\[
m_0^2 = \left[ -\Omega^2 \tan \theta I_{zz} + \Omega^2 \sin \theta \cos \theta (I_{zz} - I_{xx}) \right. \\
\left. + c_R \theta - (R \cos \theta - H \sin \theta) f_n \right]^2 \\
\left. + [d_R \Omega \tan \theta - (H \cos \theta + R \sin \theta) f_c \right]^2
\]
which has to be solved for the tilting angle \( \theta \) to obtain the behavior for steady operation.

<table>
<thead>
<tr>
<th>Table 2: Assumptions for steady state mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>assumption</td>
</tr>
<tr>
<td>( \theta = \text{const.} )</td>
</tr>
<tr>
<td>( \omega_R = \text{const.} )</td>
</tr>
<tr>
<td>( z_R = \text{const.} )</td>
</tr>
</tbody>
</table>
5. Results

Only a selection of the results of steady and unsteady operation can be presented here. In former publications as in [4] steady state behavior is discussed in more detail. Note again that all results are obtained from computations having considered pure stick between stator and rotor at the contact point. Figure 3 shows the typical torque-speed characteristic of such a motor model for a certain parameter set. The linear characteristic can be explained by looking at the differential equations again where the stiffness as well as the damping terms were modeled linearly. The only nonlinearities appear in the EULER angles and their time derivatives.

![Figure 3: Torque-speed characteristic](image)

Figure 3: Torque-speed characteristic

Figure 4 shows the start-up behavior of the angular velocity of the rotor for a loading torque of where the stick condition is satisfied (see Figure 3, $T_{load} < 0.7 \text{ Nmm}$). Note that the rotor rotates in the opposite direction of that of the wobbling motion of the stator.

![Figure 4: Angular velocity of the rotor](image)

Figure 4: Angular velocity of the rotor

Figure 5 represents the transient behavior of the angular velocity $\omega_x$. If one keeps in mind the 90° phase-shift it is clear that it also represents the behavior of the angular velocity of $\omega_y$.

![Figure 5: Angular velocity about the x-axis in the body-fixed reference frame S](image)

Figure 5: Angular velocity about the $x$-axis in the body-fixed reference frame $S$

6. Discussion and outlook

The present paper gives an overall description of a simple mathematical model recently developed, representing the steady and unsteady behavior of an ultrasonic wobbling-disk motor. All the terms from the nonlinear kinematics of this rigid body model are retained in this mathematical model. The characteristics seem to be qualitatively reliable and useful for further investigations in this kind of ultrasonic motor. Investigations on the stability of the steady mode have been made and will be published in the near future. The steady state solution is stable under the condition of pure stick between stator and rotor at the contact point. However, for certain parameter sets the solution becomes instable but not before slip appears. This will be discussed in more detail in the future.

7. References


