Further applications of the substitute-sources method

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Abstract

A substitute-sources method (SSM) was developed and tested for sound propagation in a turbulent atmosphere, including upward refraction [1]. In the present paper a situation with turbulence and a non-flat terrain is studied. A comparison with the results from a parabolic equation method (PE) shows good agreement.

1. Introduction

An outdoor acoustics situation with sound shielding by a wedge in the presence of turbulence is here studied numerically. The geometry is shown in Fig. 1, and can for instance be seen as a two-dimensional (2-D) model for a depressed road, where the effect of turbulence on the reduction of traffic noise could be of interest. The first plane of the ground, nearest the source, is $d_S = 10$ m long and slopes upward at an angle of $\theta = 20^\circ$. Thereafter the ground surface is horizontal and calculations are made up to the range $d_R = 200$ m along this plane. Here, the ground surfaces are assumed to be acoustically hard. The results from the substitute-sources method (SSM) are compared with those from a parabolic equation method (PE).

![Geometry with source, edge, substitute surface (S), and receiver.](image)

2. The substitute-sources method (SSM)

In the substitute-sources method implemented here, the sound field above the edge of the two planes is replaced by a surface, $S$, of sources (see Fig. 1). From this substitute surface there will be line-of-sight propagation to the receiver. Thereby known mutual coherence models for propagation through turbulence can be applied, and the expected sound pressure level at the receiver can be predicted. The amplitudes of the substitute sources are here found analytically as for wave propagation from an omnidirectional source above a hard flat ground. For a finite impedance ground surface or a sound speed profile, the amplitudes can be found numerically [1]. An implementation similar to the one presented here has been made for a low screen in Ref. [2], which explains the method more in detail.

In Ref. [1] a decomposition of the sound field into a coherent and an incoherent part due to a random medium was made. That way the method could use multiple substitute surfaces between the source and the receiver, which is assumed to make the method applicable to a range varying sound speed profile, ground impedance and terrain profile. The terrain profile could be modelled as step-wise linear, with a substitute surface at each edge, similar to what is done in the present paper. The decomposition into coherent and incoherent parts for a non-flat terrain will however not be studied here, but could belong to future work.

3. The parabolic equation method (PE)

The basis of the implementation of the parabolic equation method (PE) used here for the comparison with the SSM can be found in Refs. [3, 4]. The implementation is a wide-angle PE of a low order (first-order Padé expansion); using a higher order implementation gives applicability to wider angles. A Gaussian starting field and a discretisation distance of $\lambda/8$ have been used, where $\lambda$ is the sound wavelength. The turbulence effects are modelled by random realisations of the spatial distribution of refractive index fluctuations. Each realisation is found from an inverse Fourier transform using a turbu-
lence spectrum and random phases, as in Ref. [3], which is a slightly different approach from the more commonly used Fourier modes technique [5].

The application of the PE to a step-wise linear terrain was developed by Blairon et al. [6]. At each edge the calculation domain is rotated, using as starting field the result from calculating along the previous plane. In a work by Blairon measurement results are compared with calculated ones for a three-slope terrain including range variations in ground impedance and in sound speed profile [7].

4. Turbulence modelling

Here, a von Kármán turbulence model for temperature fluctuations is used, with three-dimensional spectral density

\[ \Phi(K) = AC_T^2(K^2 + K_0^2)^{-11/6} \]  

(1)

where \( A \approx 0.0330 \), \( C_T^2 \) is the structure parameter of the temperature fluctuations, \( K \) is the turbulence wave number, and \( K_0 = 2\pi/L_0 \), where \( L_0 \) is taken as the outer scale of the turbulence [8] (cf. also [9, 10]).

The corresponding mutual coherence factor (MCF) for spherical waves is evaluated numerically from

\[ C = e^{-\frac{\pi h^2}{4L_0^2} \int_0^{K_0} (K^2 + K_0^2)^{-11/6} K_0^2 K_0^2 \Gamma \left( \frac{1 - 11/6}{11/6} \right) \frac{dK_0}{2\pi}} \]  

(2)

where \( h \) is the separation distance, \( L \) the range, \( \Gamma \) the gamma function and \( K_0/d \), a modified Bessel function of the second kind. Further, \( \gamma_T \) is the extinction coefficient of the mean field due to temperature fluctuations, and can be written as

\[ \gamma_T = 3\pi^2 A k^2 K_0^{-5/3} C_T^2 \frac{L_0}{10K_0^2} \]  

(3)

where \( k \) is the acoustic wave number and \( T_0 \) the mean temperature [8].

The separation distance \( h \) of the MCF is in the SSM the distance between two sources on the substitute surface. It is assumed that the MCF for spherical wave propagation \( (C \) in Eq. 2) can be used for the 2-D modelling made here without significant errors in the calculated levels. A previous comparison for barrier insertion losses, using a Kolmogorov turbulence model, indicates that such 2-D modelling is sufficient [11].

For the PE calculations a 2-D spectrum of the turbulence is used, which is given by integrating Eq. (1) with respect to \( K_2 \), where \( K^2 = \kappa^2 + k_2^2 \), with \( \kappa^2 = k_x^2 + k_y^2 \) (see e.g. [10, App. 1]):

\[ \Phi(\kappa) = AC_T^2 \int_{-\infty}^{\infty} (K^2 + K_0^2)^{-11/6} dK_2 \]  

(4)

\[ = AC_T^2 \frac{2\sqrt{\pi}}{5} \frac{\Gamma(1/3)}{\Gamma(5/6)} (K^2 + K_0^2)^{-8/6}. \]

5. Results

The calculated results are shown in Figs. 2 and 3, for the frequencies 0.5 and 1 kHz, respectively. All results are plotted relative to the free field level as a function of the total range, \( d_S + d_R \), with \( d_S = 10 \) m. The source and the receiver heights (the normal distance to their planes) are 1.2 m and 1 m, respectively. For the PE calculations 50 realisations of the turbulence have been used to approximate the expected level. The values of the turbulence parameters are \( C_T^2 = 10 K^2/m^{2/3} \) and \( L_0 = 10 \) m, which are taken to model a strong but not unrealistically strong turbulence.

Concerning calculation times, the SSM results for a single frequency takes about as long as a single realisation in the PE method. It should however be noted that neither implementation has been optimised in this respect, and that the PE method could be applied to more
complex range varying situations, than is done here, without significantly increased computation times.

The results calculated without turbulence are compared with those from using a diffraction theory, plotted as dashed lines in the Figures (uniform theory of diffraction, UTD; see e.g. Ref. [12]). The agreement with the SSM and PE results is good: around 0.2 dB deviation at most. (The SSM results are plotted with bullets and the PE results as a solid line. The lower curves are without turbulence.) The deviation is expected to grow stronger if the diffraction angle is increased. This is due to that one uses, in essence, a kind of Kirchhoff approximation in both the SSM and in the PE, i.e. the starting field above the edge is approximated by the incoming free field. It should be noted that the small oscillations in the PE results without turbulence at 1 kHz (Fig. 3) are not physically correct but are due to numerical errors.

The upper two curves at longer ranges show the results including turbulence. The agreement between the SSM and PE results is good. The smooth SSM curves seem to well follow the mean of the more noisy looking PE curves. For the 0.5 kHz calculations (Fig. 2) the increase due to the turbulence is about 1 dB, and for the 1 kHz calculations (Fig. 3) the increase is about 2 dB.

6. Conclusions
Results from a substitute-sources method (SSM) have been compared with those from a parabolic equation method (PE) for a situation with a wedge in a turbulent atmosphere, at sound frequencies of 0.5 and 1 kHz. The good agreement between the SSM and the better established PE method indicates that they both can be applied to problems like this, with a step-wise linear terrain and atmospheric turbulence. The increase due to the turbulence was here about 2 dB at most, at 1 kHz, and is expected to increase at longer ranges.

7. Acknowledgments
The research behind this paper has been financially supported by a Marie Curie Fellowship of the European Community programme Energy, Environment and Sustainable Development under contract number EVK4-CT-2001-50004.

8. References


